34th Annual INCOSE international symposium
hybrid event
Dublin, Ireland
July 2 - 6, 2024

Innovation Ecosystem Dynamics, Value and Learning I: What Can Hamilton Tell Us?

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Abstract. Held in Dublin, Ireland, IS2024 invites us to refresh understanding of contributions to systems engineering by Ireland's greatest mathematician--Sir William Rowan Hamilton (1805 - 1865), Professor of Astronomy at Trinity College Dublin and Royal Astronomer of Ireland. His profound contributions to STEM deserve greater systems community attention. Supporting theory and practice, they intersect Foundations and Applications streams of INCOSE's Future of Systems Engineering (FuSE) program. Strikingly, key aspects apply to systems of all types, including socio-technical and information systems. Hamilton abstracted the energy-like generator of dynamics for all systems, while also generalizing momentum. Applied to the INCOSE Innovation Ecosystem Pattern as dynamics of learning, development, and life cycle management, this suggests an architecture for integration of the digital thread and machine learning in innovation enterprises, along with foundations of systems engineering as a dynamical system.

Keywords. Digital Thread; Hamiltonian; Hamilton's Principle; Energy; Momentum; Machine Learning; FuSE; Future of Systems Engineering; Foundations; Organizational and Social Systems Modeling.

Introduction

This paper highlights contributions William Rowan Hamilton made to the theoretical foundations of scientific and engineering disciplines, and some current questions to which they could apply. Hamilton's mathematically-based patterns describe the phenomena of mechanics, electrical science, thermodynamics and subsequent disciplines, supporting the foundations of today's STEM. However, in the general setting of systems engineering, over-limiting assumptions about applicability sometimes arise. This paper briefly recalls aspects of Hamilton's contributions, and why current assumptions may be unnecessarily limiting practitioners. Prominent examples of current interest are noted—information systems and socio-technical systems of innovation. These suggest architecture-level strategies for integrating the digital thread and machine learning into the innovation enterprise. This is described in the perspective of the INCOSE Innovation Ecosystem pattern, which provides a general descriptive reference representation of enterprise or supply chain engineering and life cycle management processes, as a system of systems in its own right. This reference pattern interprets "innovation" very broadly, as including the entire life cycle of all products and systems, whether they are effective or not, providing a neutral framework for analysis use.

Millennia of observation and thought about natural phenomena were punctuated by a much shorter revolution. In less than 300 years, Newton, Lagrange, Gauss, Euler, Jacobi, Hamilton, Gibbs, and many others synthesized, extended, refined, and applied conceptual and mathematical frameworks that supported the dramatic acceleration of STEM. What followed rapidly changed the quality, length, and possibilities of human life.

Those mathematical frameworks provided conceptual and quantitative models to describe, predict, or explain many aspects of the modeled world, deterministic and probabilistic. Hamilton's contributions were recognized by later thought leaders as remarkably universal across phenomena of mechanics, electrical science, and other fronts. Max Planck (1858-1947) noted that "The chief law of physics, the pinnacle of the whole system is, in my opinion, the principle of least action"—Hamilton's Principle (Planck, 1925).

Contemporary Innovation Ecosystem Questions

Systems engineering today frequently involves (1) information systems and (2) socio-technical systems. It is increasingly common for engineered products to directly involve these domains, and even more common for the engineering enterprise itself to depend upon them. Even though they were not the main interest in Hamilton's time, today these domains have rapidly growing significance for systems engineers.

Related engineering project questions that Hamilton's contributions may help us answer include:

A. <u>Project and program planning</u>: What are predicable efforts, times, and costs of performing innovation and life cycle management? What are related uncertainties (and consequent risks) in those predictions? These are questions addressed historically by empirical models such as COSYSMO (Valerdi, Boehm, & Reifer, 2003) and more recently asked by INCOSE FuSE Foundations efforts (de Weck, 2023). They are supported by basic shared understandings of enterprise processes such as (ISO, 2023) and (Walden et al, 2023). When projects involve complications of organization (such as supply chains or consortia), problems with communication, incentives, shared understandings, or cultures, their success may be doomed before execution begins.

B. <u>Project execution management</u>: As projects are performed (and encounter real-world perturbations only partly predictable), what are the means of preparing, monitoring, and directing them for optimum outcome—including decision-making in particular? During complex multi-enterprise development projects, how can we detect and act on systemic project uncertainties and instabilities threatening success? These are questions addressed historically by disciplines such as capabilities assessment (SEI, 2010), project management in general (Rebentisch, 2017), agile methods in particular (Dove, 2001), and emerging aspects of digital engineering (Schindel, 2022).

C. <u>Project learning and its recurrent application</u>: What are means and effects of accumulating new experience in items (A) and (B) above, and effectively distilling, managing trust in, and applying knowledge and competency in future projects? This question is addressed historically by technical readiness levels (Mihaly, 2017), capability maturity models (SEI, 2010), knowledge management (Trees, McCulloch, Witt, 2021), application of recurrent patterns (Alexander, 1977), (Gamma et al, 1994), (Cloutier 2008), (Schindel, 2022), and product line engineering (Clements & Northrop, 2002) (ISO, 2021). It includes the emerging subject of machine learning (LeCun, Bengio, & Hinton, 2015).

D. <u>Information and information system roles in items (A), (B), and (C) above</u>: A common thread through the above are roles of information and information systems—both those using engineered information technologies and those performed by human beings. What is the theoretical basis for engineering the performance of these subsystems as an integrated part of the larger enterprise systems in which they appear? How can systems engineering connect these? These questions are addressed historically by information theory (Shannon, 1948), enterprise architecture (Foorthuis, Steenbergen, Brinkkemper & Bruls, 2016), digital engineering (Schindel, 2022), the digital thread (Cribb et al, 2023), and machine learning (LeCun, Bengio, & Hinton, 2015). More recently, US and European governments are issuing executive orders and regulations demanding new levels of mastery of what is emerging.

What can Hamilton tell us about the above questions?

A Challenge to Contemporary Assumptions

Hamilton's framework may be most familiar to engineers in mechanical, civil, or electromagnetics settings. The systems community may be assuming that Hamilton's mathematical contributions do not address the socio-technical and information system questions above in a practical way.

One sign of such an assumption in the INCOSE and other systems communities is a continued call and search for what are perceived as missing theoretical foundations for the science and engineering of generalized systems (Friedenthal et al, 2021). Disciplines in engineering and sciences are concerned with phenomena (e.g., mechanical, electrical, chemical) specific to those disciplines, leading to impactful phenomena-specific patterns of interactions described by laws specific to those disciplines, often in mathematical form. What about equivalent impactful phenomena, theory, and mathematics for systems in general?

A counter-argument is that more attention should be given to already modeled phenomena (from Hamilton and other STEM pioneers) before spending too much effort looking elsewhere (Schindel, 2016; 2020). Three such phenomena have been suggested, playing parts in this paper: (1) the System Phenomenon, studied by Hamilton; (2) the Value Selection Phenomenon, fueling innovation force; and (3) the Group Learning and Trust Phenomenon, learning and applying patterns in the face of uncertainties.

We do not suggest that unnoticed phenomena and laws concerning information systems and socio-technical systems are not waiting for discovery. However, as already noted by those who followed him, Hamilton's framework is not limited to only mechanical or other specific phenomena.

Informal Summary of the System Perspective Informed by Hamilton

(Hamilton, 1834) showed we can describe energy (or at least an energy-like characteristic function) of a system in a general and mathematical way not restricted to only some systems. Hamilton and those who followed showed how deeply these concepts follow from the most limited set of ideas present in many systems—even seemingly "soft" systems. Only the concepts of system interaction and state are required to get started. An informal argument proceeds as follows:

A. <u>Systems</u>: Start with a system <u>of any type</u>. By "system", we mean a set of interacting system components (Figure 1). By "interact" we mean they exchange input-outputs, such as force, material, energy, or information, resulting in changes of state of the components. By "state" of a component we mean the condition of the component that can modify its current input-output behavior. Interaction thus changes state, which in turn impacts interaction.

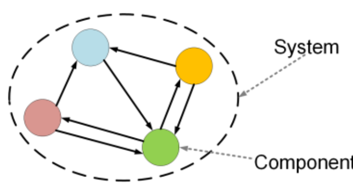


Figure 1: The System Perspective

B. <u>Non-deterministic and Discrete Systems</u>: This short and informal discussion focuses on deterministic, smoothly continuous systems. However, it turns out that <u>discrete</u> Hamiltonian systems have been heavily explored and exploited, including providing the symplectic Hamiltonian integrators found in numerical simulation (Shibberu, 1994), (Marsden & West, 2001). For <u>non-deterministic</u> cases, Hamiltonian mechanics provide the foundations of the rich historical field of statistical mechanics (Gibbs, 1901), (Khinchin, 1949), where state flows are replaced by probability density flows. Probabilistic cases also re-enter this story through machine learning and human behavior.

C. States: Have a way of representing the state of the system of interest Q(t) = $\{q_1, \ldots, q_n\}$, whose values change over time at rate $\dot{Q}(t) = \{\dot{q}_1, \ldots, \dot{q}_n\}$, believed sufficient to characterize observed interactions.

D. Characterizing System Level Behavior: Imagine now a scalar-valued function of state and time, *not yet defined until below*, contributed by Hamilton: H(Q, $\dot{Q}$, t), intended to characterize something about the system--we have not said how yet.

E. Generalized Momentum: Hamilton contributed a "generalized momentum", P(t) = $\{p_1, \ldots, p_n\}$, intended to generalize the idea of momentum in elementary physics--describing ability to change $\dot{Q}$. His generalized P is defined by the sensitivity of H (H not otherwise defined yet) to $\dot{Q}(t)$:

$$p_i \equiv \frac{\partial H}{\partial \dot{q}_i} \tag{1}$$

(Notice that if H turns out to be something "like" energy, this says that momentum is the sensitivity of energy to changes in velocity, or that energy is required to change velocity, an intuitively reasonable generalization of mechanical systems.)

F. Defining the Hamiltonian: We want H to characterize the system's (Q,P) trajectories, and will do so here by tying them to the local slopes of surface H. (See Figure 2.) First, the local sensitivity of H with respect to p_i at (Q,P) is to be equal to the time rate of change of q_i along the system state trajectory passing through (Q,P):

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \tag{2}$$

Second, the local sensitivity of H with respect to q_i at (Q,P) is to be the negative of the time rate of change of p_i along the system state trajectory passing through (Q,P):

$$\dot{p}_i = - \frac{\partial H}{\partial q_i} \tag{3}$$

For intuition, notice that dividing both sides of Eq (2) by both sides of Eq (3) shows that the instantaneous direction of motion in the (q, p) plane of Figure 2 is the same as the ratio of the local slopes of H in the q and p directions.

The above reasoning is important to intuitive motivation and perspective on applying Hamiltonians. Hamilton took the major step of providing Eq (1) as a definition of generalized momentum, but defined H through a Legendre transformation of a pre-existing Lagrangian, which we are not assuming here, as we are making no assumption of pre-existing energy concepts. A traditional textbook perspective is to start with a ***mechanical*** system having defined kinetic and potential energies and a Lagrangian, then applying a Legendre transformation to yield a Hamiltonian that is based conceptually and mathematically on mechanical energy (Greenwood 1977) (Landau and Lifshitz 1976). In that reasoning path, one then proves that Eq (2) and (3) follow. Here, we instead define H as a function satisfying Eqs (1), (2), and (3), for a collection of actual trajectories, whether known or unknown. The mathematical question of existence (not all systems are Hamiltonian) is informally addressed in item (H) below.

G. Hamilton's Equations: Equations (2) and (3) are Hamilton's Equations for the time evolution of the state of the system—they are stated as equations of motion, describing trajectories in terms of H. It may seem odd that we have arrived at the equations of motion of a system, but we do not know what specific kind of system it is yet! Intuitively, this is because we started with a set of trajectories and invented a real-valued function of state that characterizes those trajectories. See Figure 2.

H. A "Story Experiment": To ground ourselves in both intuitive and practical framing of Hamilton's Equations, here is a related "story" experiment that in recent years has been repeatedly performed by multiple parties for different types of systems, with variant approaches including (Bertalan, 2019), (Greydanus, 2019), (Toth, Rezende et al, 2020), (Bhat, 2020), (Chen and Tao, 2021):

i. Identify a specific system of interest, of any type, that you can directly observe.

ii. As the system operates, observe and record a series of $(Q, \dot{Q}, t)$ state trajectory tuple samples.

iii. Set up a machine learning (ML) system to "learn" (discover) a functional surface $H(Q, \dot{Q}, t)$ that minimizes across the sample space the following learning loss functional:

$$\text{Loss}[\, H(Q, \dot{Q}, t)\,] = \sum \left\{ \left(q_i - \frac{\partial H}{\partial p_i} \right)^2 + \left(\dot{p}_i + \frac{\partial H}{\partial q_i} \right)^2 + \left(p_i - \frac{\partial H}{\partial \dot{q}_i} \right)^2 \right\} \qquad (4)$$

iv. Two terms of Eq (4) show the "learned surface" attempts to satisfy Hamilton's Equations (2) and (3) for the observed training data. The third term attempts to satisfy (1) to discover generalized momentum. Thus, we can "discover" a Hamiltonian surface from observational data.

The main point of this "story experiment" is not machine learning—it is that Hamilton provided a conceptual function H that *characterizes the dynamic behavior of any system having deterministic continuous state trajectories* (see also B, F above), by how the function H defines a "map" of state trajectories. H is a characterization of the system, sometimes referred to as a "generator" of the system's dynamics. In any neighborhood of the (Q, P) plane where we have a set of observations, those observations can provide estimated time rates of change for Q and P along the system's state trajectory. From that, Hamilton's equations effectively define H quantitatively (up to an additive constant) by telling us about the local slopes in the surface of H at those points with respect to Q and P axes. See Figure 2(a). Knowing nothing except the observed trajectories of the system, we have created the surface H, which thereby characterizes that system's behavior (as trajectories in the Q,P plane).

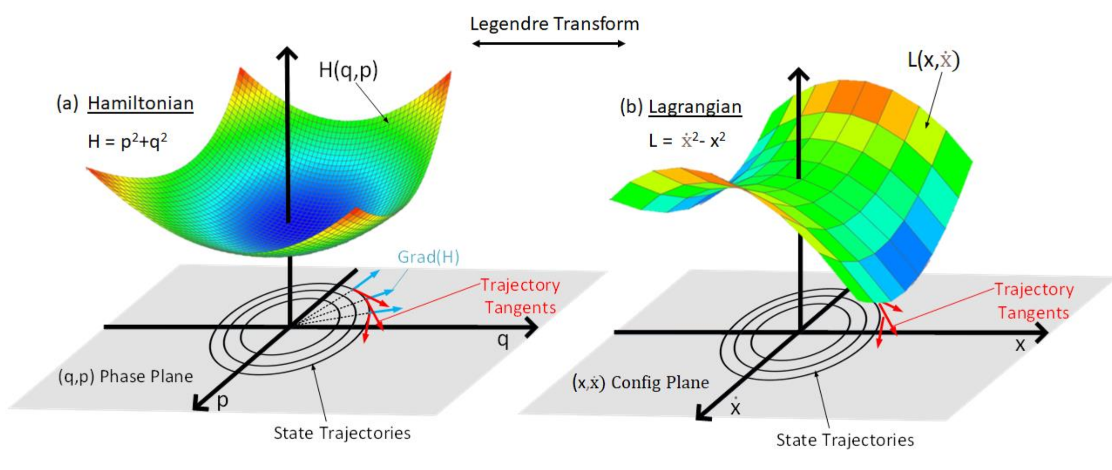


Figure 2: Phase Plane, Hamiltonian; Config Plane, Lagrangian—for Simple Harmonic Oscillator

I. The Variational Version: In Figure 2(a) a trajectory of the system occurs in the (Q, P) phase plane. At a given point on that trajectory, its direction in that plane is the tangent vector $\{ \dot{Q}, \dot{P} \}$. The gradient of H in the (Q,P) phase plane is the vector $\{ \frac{\partial H}{\partial Q}, \frac{\partial H}{\partial P} \}$, pointing in the direction of maximum rate of change of the surface H(Q,P). In that plane, and perpendicular to the gradient, is vector $\{ \frac{\partial H}{\partial P}, -\frac{\partial H}{\partial Q} \}$, pointing in the direction of zero rate of change (constancy) of the surface H(Q,P). But based on Hamilton's Equations above, that is the same as the trajectory tangent vector, $\{\dot{Q}, \dot{P}\}$. So, the trajectory of the system moves in the direction of zero rate of change of the surface H. H is thus invariant (constant, conserved) in time along its trajectory, *by the very definition of H*. Figure 2(b) shows the Lagrangian surface L for the same system. It expresses the variational statement of Hamilton's Principle (Lanczos, 1986), noted by Max Planck as remarkably broad. Here, we see that it can apply to many systems for which we can define states, including information systems and socio-technical systems.

J. Holonomic? Conservative?: The proposed potential energy concepts described in the next (application) section suggest that the systems of interest described there for Hamiltonian treatment are holonomic. In the main dynamics implied for such information and socio-technical systems, H there appears possibly conserved, whether or not it is called "energy". See also "Dissipation" later below.

Application: System States and Learning in an Innovation Ecosystem

The above discusses the dynamic evolution of system state variables Q(t). But what are the practical, real project state variables that we care about for the enterprise information and innovation project questions listed earlier above? The following sections focus on some key state variables.

Ecosystem States Associated with Learning

The American Institute of Aeronautics and Astronautics published AIAA's Digital Thread Reference Model (Cribb et al, 2023). The core of this AIAA reference model is based on the INCOSE Agile Systems Engineering Life Cycle Management (ASELCM) Pattern (also known as the Innovation Ecosystem Pattern) (Schindel and Dove 2016). A central theme of these reference models is the paradigm of "consistency management" (Schindel, 2021), which seeks to manage over the duration of a project the reduction (ideally, to zero) of a set of managed consistency "gaps" that are familiar in the history of engineering and life cycle management projects, and which run through the backbone processes of (ISO, 2023) and (Walden et al, 2023). A few prominent examples of the long list of consistency issues are:

- Is the product design consistent with its requirements?
- Are those requirements consistent with the mission and stakeholder needs and priorities?
- Are the emergent behaviors (both required and to be avoided) in the engineered system consistent with the experience about the underlying phenomena from which they emerge?
- Are instances of the manufactured product consistent with the design specifications?
- Is the observed use of the product consistent with the product mission and requirements?
- Is performance of the deployed product consistent with the specified requirements?
- Is the environment of use of the product consistent with its representation in the product mission and requirements?

Reducing these and other consistency gaps generates learned information. Learning occurs over the course of projects, much of it by humans, with some of it captured in artifacts and some in tribal knowledge. In current and future projects, more of this learning includes digital engineering agency that is only partly human, with more learning captured in digital artifacts.

The earlier list of project questions and the above consistency management paradigm now help us see a project as two kinds of mathematical boundary value problems:

- **Boundary Value Problem 1--The web of end-state consistencies**: Figure 3 illustrates the idea that a product design, implemented, delivered and in service, reflects selection pressures to minimize a set of consistency gaps. Visualize equilibrium "relaxation" of the springs into their "trade off" positions during a project. Project time is not explicitly visible in this view, although some of the consistencies it shows may themselves be about project time.

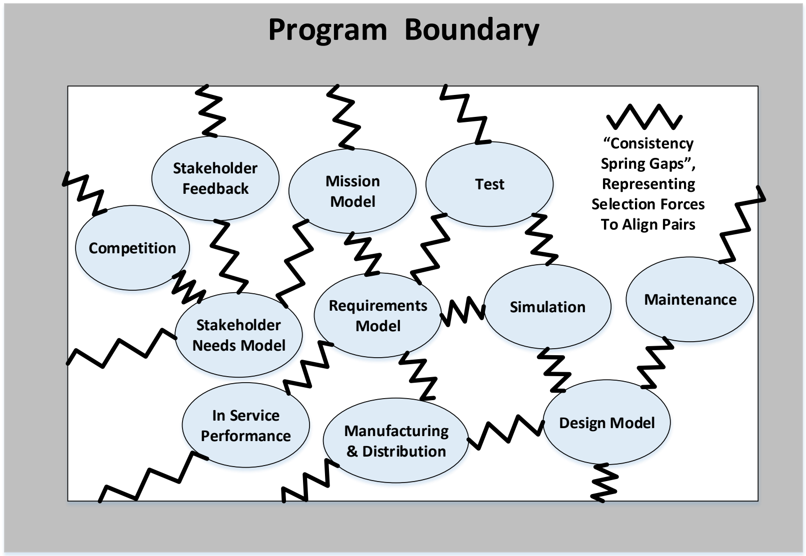
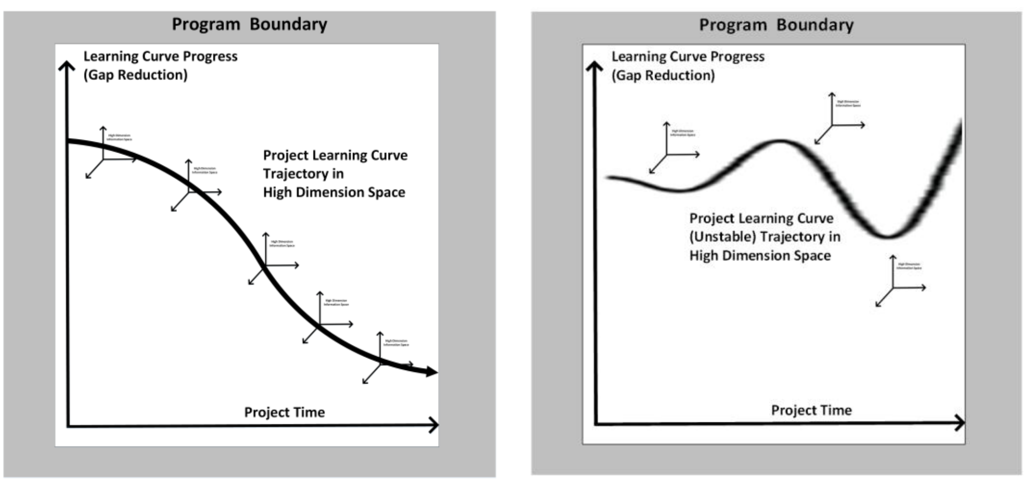


Figure 3: Example Consistency Gap Web of Elastic Springs—A Metaphor

- **<u>Boundary Value Problem 2—The dynamics of state evolution over the project duration</u>**: In contrast to that end state view, Figure 4(a) illustrates the idea of what occurs <u>during</u> a project, as a dynamic trajectory in high dimension space, progressing over time. (One might visualize the elastic network of Figure 3 "vibrating" and "relaxing" during this time.) Analogous to a control system boundary value problem, this perspective is more about questions concerning the dynamic behavior of the project itself over time, as a dynamical system.



4(a): Well Behaved Learning **4(b): Ringing, Unstable**

Figure 4: Example Learning Curve Trajectories for a Project

We are not guaranteed that an actual dynamic project system will be well-behaved, converging to a deliverable good outcome. It may "ring" or become unstable, illustrated by Figure 4(b). Hamilton's contribution of "generalized momentum" (discussed in the previous section) ultimately figures into this.

Figures 3-4 illustrate that, for a development and life cycle management project, it is the states of the managed consistency gaps that we should especially care about as candidates for the ecosystem state trajectory model in the Hamilton perspective. Further, this trajectory can be seen as the innovation ecosystem learning the information necessary to reduce inconsistencies to deliverable levels.

This also prepares us to differentiate between what was already known at the start of the project (*a priori* knowledge; "priors"), versus what is learned during the project. That differentiation is central to the practical integration and management of learned formal patterns expressed as parameterized models, along with more informal tribal knowledge and heuristics. It is also central to application of Bayesian inference (Jaynes, 2003), dramatically successful in communication and navigation systems.

The Level 1 (Figure 5) view of the ASELCM Pattern (Schindel & Dove, 2016) (Schindel, 2022) incorporates that differentiation, showing:

1. Life Cycle Management for System 1 acts based on *what is already known* about System 1 and its environment;
2. Learning & Knowledge Management for System 1, for learning new information about System 1 and its environment (whether human-based learning, machine learning, or their combination).

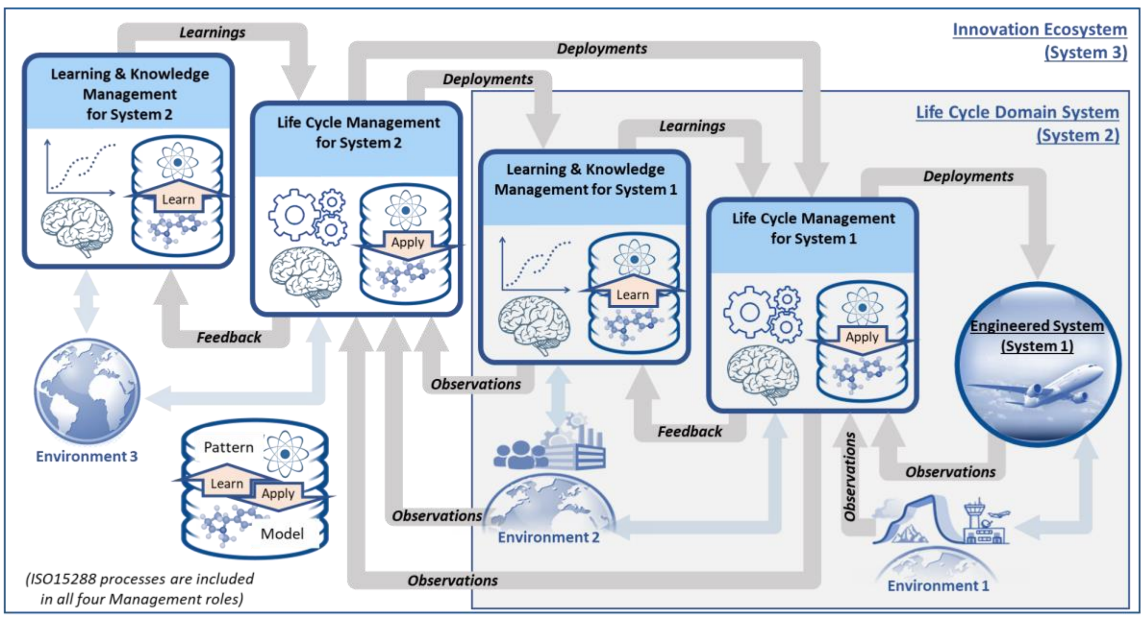


Figure 5: ASELCM Pattern, Level 1 View, Separating Learning from Application
(Adapted from (Cribb, 2023))

The ASELCM Level 2 (Figure 6) view shows:

1. the already learned Deployed Generic Model (Pattern), more general than needed for the specific project, hence to be configured;
2. the configured Specific Model, specific for the project.

This reference model is not to say that human enterprise project teams always respond optimally, but rather to study the forces to which they respond, by representing the perceived loss functions. These can also be central to automated machine learning algorithms.

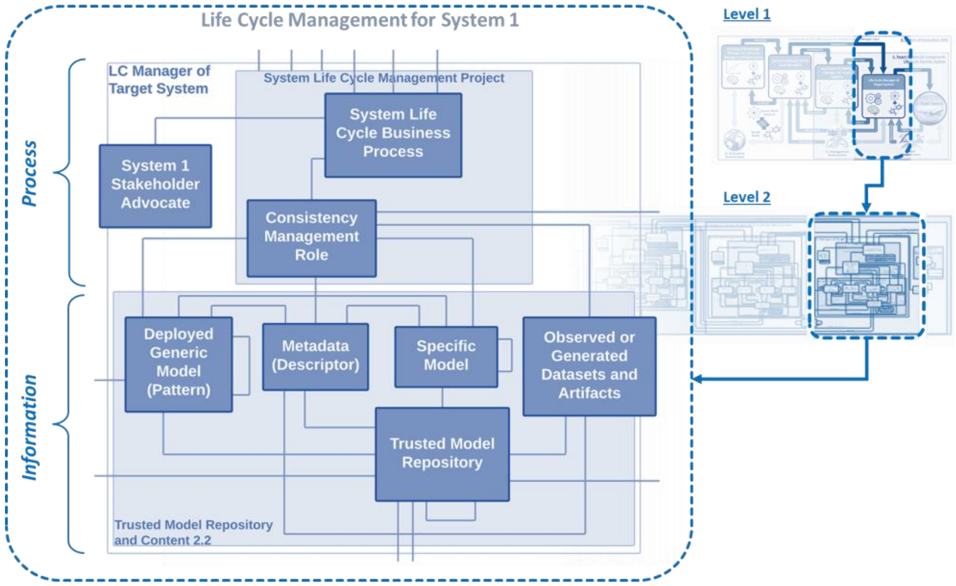


Figure 6: ASELCM Pattern, Level 2 View, Information versus Processes
(Adapted from (Cribb, 2023)

Machine Learning in the Ecosystem

Public awareness of machine learning progress has recently grown dramatically. However, it results from 75 years of efforts across dramatically improving methods, along with orders of magnitude advance in hardware and training data resources (LeCun, Bengio, & Hinton, 2015).

Central to the contemporary machine learning work is the concept of minimization of some form of loss function by various training algorithms. Hopfield's seminal PNAS paper of 1982 (Hopfield, 1982), reawakening artificial neural network interests, described minimization of what he referred to as an "energy" function. Inspired at the time by properties of both biological neural networks and physics of dynamical system state flow patterns, Hopfield referred to results as "isomorphic with an Ising model" of physics.

In the more recent efforts (LeCunn, 2006, 2021), "energy-based methods" have become popular in machine learning. The continued reference to "energy" in this work stems from recognition of the deep connection between probability distributions governing the performance of neural nets over large sample spaces and the probability distributions of statistical physics (e.g., Gibbs-Boltzmann distribution; Helmholtz Free Energy Distribution)(Hinton and Zemel, 1993).

The State Variables

As illustrated above, the key "project state variables" we want to manage effectively (possibly with help from Hamilton) include the consistency gap signals. These contribute to the "potential energy" (Q related) part of the Hamiltonian, as they describe the "consistency gap field" that any project seeks to minimize through selection forces. However, these are not the only state variables, as the metaphor of Figure 3 is replaced by inter-role selection force interactions in Figure 6; its Consistency Management Roles and Business Process Roles contribute additional state variables further characterizing the organization's processes, capabilities, and culture.

Real Projects: Decision-Making and the Digital Thread

Executing a project involves *making decisions*. Some of these decisions are high-visibility major choices by senior decision-makers, at major stage gates. Many other decisions occur across the teams on a day-to-day basis. With the above consistency management paradigm, we can think of those decisions across the system life cycle in an additional way: *All program decisions are reconciliations of inconsistencies* (Schindel, 2023, 2024).

The digital thread reference model (Cribb et al, 2023) represents roles of detection of inconsistencies (by human or automated agents) and reconciliation of those inconsistencies (more likely by human agents, potentially with future automated assistance). Snapshot records of the related information items form a “consistency thread” precursor of the digital thread of Figure 7.

In the innovation system dynamics, the resulting consistency thread/digital thread plays these major roles:

1. From a system dynamics perspective, it is a trace of the project state trajectory of Figs 3 and 4.
2. It is also the record of detected inconsistencies, and their reconciliations.
3. It exposes data for use in learning. Whatever the project outcome, it provides a learning database, for human or machine learning.
4. It provides support for the use of past learning.

Figures 5, 6, and 7 summarize aspects of the architectural pattern for integration of the enterprise, the digital thread, and human and machine learning.

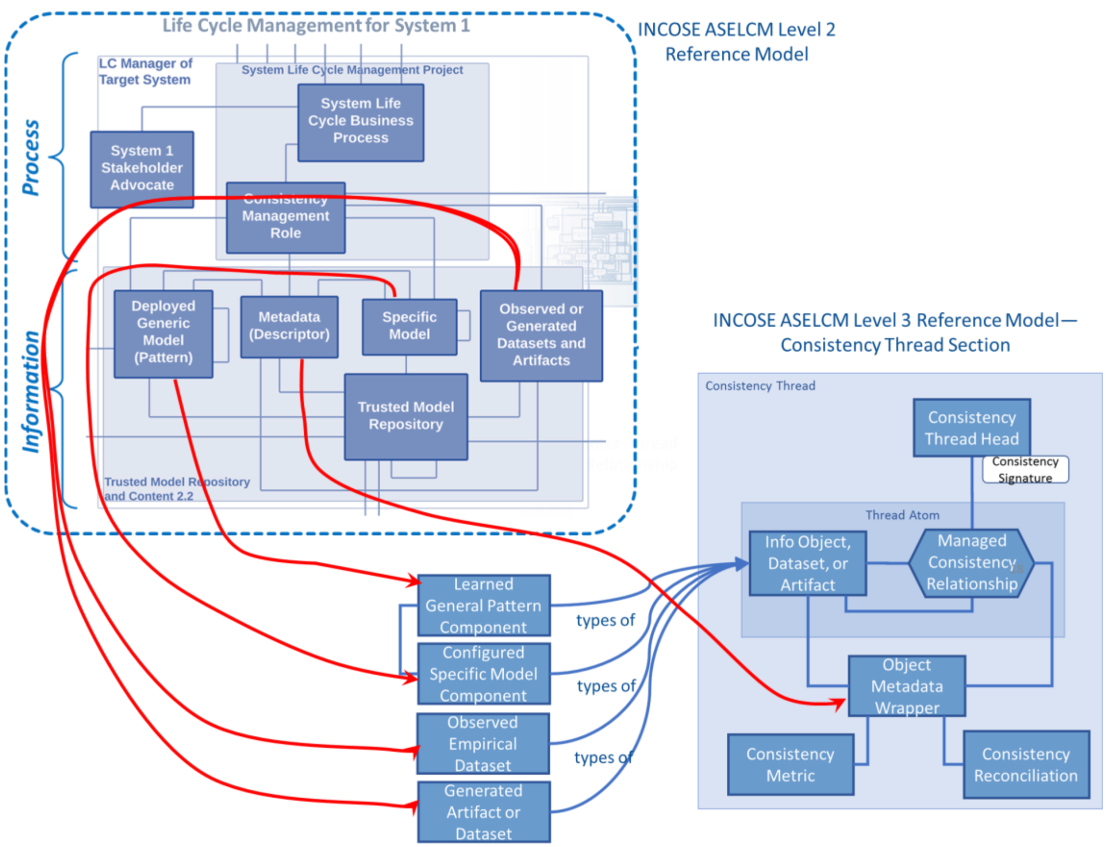


Figure 7: ASELCM Pattern, Consistency Thread View
(Adapted from (Cribb 2023))

Are Classical Physics Models Practical for Socio-Technical Systems?

The question of treating execution of complex, risky innovation projects as a mathematical problem of optimal control was considered in (Schindel, 2017). However, it is reasonable to question whether using Hamilton’s mathematically-based model is practically plausible for complex, human-performed socio-technical systems such as engineering and life cycle management. Do differential equations really have any practical place here? Similarly, is it reasonable to expect that machine learning can be productive in this human judgment-intensive technical context?

That such questions would even be seriously considered has recently become more likely, based on advancements leading to surprising demonstrations, such as machine learning informed algorithms passing legal bar and medical licensing examinations or performing diagnoses. How is it that a machine learning algorithm based on Jacobian matrices of partial derivatives and flowing with numbers has led to such capabilities? While the answers are emerging, clearly earlier intuition about the limitations of mathematics of classical mechanics in this space needs to be recalibrated now, because of demonstrated progress in performance enabled by better algorithms, accessible training data, and hardware capacities.

At the very least, this encourages preparatory re-acquaintance with Hamilton's pattern. Even currently human-intensive cases begin to illustrate the enterprise architecture into which advanced versions of the digital thread can be integrated for enterprise learning. The discrete time and statistically-based versions of Hamilton's pattern are likely to be the most relevant for the Innovation Ecosystem—but that is already the case for much of contemporary engineering's use of Hamilton's contributions.

If we hope to apply the methods of optimal estimation and control in the presence of randomness and uncertainty (they have been very successful for simpler engineered systems) to the system of engineering and life cycle management itself (Schindel 2017), then we first need to have a theoretical representation of that system. Likewise, if we want to have a theoretical basis for understanding the behavior of autonomous learning and inference algorithms of artificial intelligence (Cribb et al 2023), then we need sufficient representation of them as dynamical systems. It appears that Hamilton and those who followed have provided us with such a representation, if we reason in the right order.

Ecosystem Selection Forces, Dissipation, Entropy, and Complexity

The above application discussion focused on potential energy in the Innovation Ecosystem, but the selection forces provided by other ecosystem roles (Schindel, 2020, 2023, 2024) contribute kinetics to the dynamical behavior of this system. For discussion in a subsequent paper, certain aspects beckon:

1. Dissipation is about reversibility. As learning proceeds in an Innovation Ecosystem project, the potential energy associated with consistency gaps shrinks macroscopically at the ecosystem level. *If* the ecosystem is to conserve H, what (kinetic; potential) would grow to offset that shrinkage? An interesting candidate is the project's digital thread information, captured during learning to "explain" (and defend for posterity) a learned product model's validity as a compression of empirical data. At a more microscopic level, Landauer, Bennet, Feynman, Toffoli and others have pursued the concepts of dissipation-free information processing, with the exception of dissipation by erasure. (Hey, 1996).
2. Hamiltonian systems also conserve information entropy (Carcassi and Aidala, 2020). Using Kolmogorov definition of complexity as size of the generator (Li and Vitany, 1997), and recalling Shannon entropy's connection to encoded message size, may imply a form of conservation of complexity of the Engineered System in the ecosystem (de Weck, 2023). However, note that complexity of what the Ecosystem of Figure 5 must learn in a project is not the full complexity of the Engineered System, but of the "posterior" aspects of it—separating "what do we *already* know?"
3. The learning subsystem of the ecosystem can be Hamiltonian (Ramacher, 1992)

4. In the study of dynamical systems, a long and rather complex history of research dating to Hertz in 1894 has described the nature and consequences of non-holonomic constraints. (Bloch, 2003), (Rojo and Bloch, 2018), (Flannery 2005), (Eden, 1951).

Conclusions and Future Work

This synthesis paper has:

1. Outlined some of the strategic questions faced by contemporary Innovation Ecosystem projects;
2. Provided an informal refresher on how Hamilton's framework can apply to diverse systems, including socio-technical and information systems, and to the Innovation Ecosystem in particular;
3. Shown that the key Innovation Ecosystem state variables relevant for Hamiltonian "potential" modeling include the ASELCM Pattern Consistency Management "gaps" central to the digital thread;
4. Noted that Energy Based Learning methods for machine learning algorithms are already being used to learn real system Hamiltonians as well as being Hamiltonian modeled themselves;
5. Shown that consistency management's needs for inconsistency detection and reconciliation are candidates for machine learning based aids to traditional labor-intense roles;
6. Shown that this synthesis suggests an Innovation Enterprise architecture integrating the digital thread as well as machine and human learning;
7. Laid a foundation for future momentum kinetics and applications work utilizing these approaches, as well as case study work.

Related work continues to progress in the INCOSE Patterns Working Group, supporting the INCOSE FuSE initiative, and additional collaborations with other working groups, societies, and enterprises.

Acknowledgements

We have to thank W.R. Hamilton for the core mathematical insights described here--even if we must rediscover them. Rick Dove led the INCOSE Agile SE Discovery Project during which the ASELCM Pattern described here was applied and advanced. When the ASELCM Pattern was used as the basis of the AIAA aerospace digital thread reference model, Woong Je Sung contributed the friendlier but still correct diagrams of the original SysML pattern shown here as Figures 5, 6, and 7, for general audiences.

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